Effects of telescope secondary mirror spider on optical astronomical imaging

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Master of Science in Astronomy

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Abstract

The spiders hold the secondary mirror and control the mirror from vibrations, and also allow the mirror to be tilled. The diffraction of spiders introduce spikes in the image of the point spread function (PSF).

The literatures have not covering this area very well and there are very few papers concerning imaging with spider apertures. The work in this thesis demonstrate the effects of these spiders on the actual images of optical astronomical telescope.

First the effects of spider diffraction are studied in terms of imaging a point source. This involves the Strehl ratio, FWHM of the PSF, the fractional encircled energy, and the MTF.

The study is also extended to examine these effects on the power spectrum of binary stars.

The results show that using curved spiders, no significant effects occur to the diffraction pattern of the aperture.

Finally it should be pointed out here that all the simulations and the results are carried out with MATLAB v7.1
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List of Acronyms

NASA        National Aeronautics and Space Administration.
ESA         European Space Agency.
CSA         Canadian Space Agency.
HST         Hubble Space Telescope.
NGST        Next Generation Space Telescope.
VLT         Very Large Telescope.
MACO        Multi-Conjugate Adaptive Optics.
AO          Adaptive Optics.
PSF         Point Spread Function.
NIR         Near Infrared.
ASM         Adaptive Secondary Mirror.
OTF         Optical Transfer Function.
MTF         Modulation Transfer Function.
FWHM        Full Width at Half-Maximum.
RPS         Ratio of the secondary peak power spectrum to the primary peak power spectrum.
Chapter One

Introduction

1-1 Optical Telescope

Ever since Galileo first pointed a simple refracting telescope in 1609, astronomers have wished for higher and higher resolution imaging instruments. Early telescopes were limited by the accuracy with which large lenses could be figured. The development of reflecting telescopes by James Gregory and Isaac Newton lead to a rapid increase in the resolution available to astronomers. With the work of Thomas Young in the 19th Century, astronomers realized that the resolution of their telescopes was limited by the finite diameter of the mirror used. This limit was set by the wave properties of light and meant that large, accurately figured mirrors would be required in order to obtain higher resolution. Well figured telescopes with larger aperture diameters were constructed, but the improvement in resolution was not as great as had been expected. The resolution which could be obtained varied with the atmospheric conditions, and it was soon realized that Earth's atmosphere was degrading the image quality obtained through these telescopes.

For much of the 20th Century, the blurring effect of the atmosphere (known as atmospheric "seeing") limited the resolution available to optical astronomers. This degradation in image quality results from fluctuations in the refractive index of air as a function of position above the telescope. The image of an unresolved (i.e. essentially point-like) star is turned into a dancing pattern of "speckles". In order to obtain better atmospheric seeing conditions, telescopes were constructed at high altitudes on sites where the air above the telescope was particularly stable. Even at the best observatory sites the atmospheric seeing
conditions typically limit the resolution which can be achieved with conventional astronomical imaging to about 0.5 arc seconds at visible wavelengths.

NASA and ESA – joined by the Canadian Space Agency (CSA) – have since 1996 collaborated on the definition of a successor to the Hubble Space Telescope (HST). This successor, the Next Generation Space Telescope (NGST) is foreseen to be a passively cooled, 6.5m aperture class telescope, optimized for diffraction-limited performance in the near-infrared (1 – 5 µm) regions, but with extensions to either side in the visible (0.4 – 0.7 µm) and mid-infrared (5 – 28 µm) [1].

More than a decade ago there were approximately ten 4-m class ground based telescopes in operation with one 10-m telescope coming into service. Today there are perhaps 12 telescopes in the 4-m class while, remarkably, no less than 16 telescopes in the ~8-m (Very Large Telescope (VLT)) class will soon be coming into operation. In space the now-veteran 2.4-m HST continues to generate a steady stream of dramatic results while the propose of its successor, the NGST, plan to hurdle the 4-m category completely and, at 8 meters aperture, to begin an era of space VLTs.

On the ground, meanwhile, the advents of the VLTs, and the prospect of NGST in space, have not stopped the progress of ambition. Even in the early 90's consideration of possible 25-m class ground-based facilities was developing and technical studies progressing. Momentum has increased; the possibility of 100-m telescopes was soon explored. Even more recently, the critical technique of Multi-Conjugate adaptive optics (MCAO) has been demonstrated to be practicable. These promises are to liberate adaptive optics (AO) from the limitations of classical single-guidestar techniques, which constrain fields of view to a few arc sec, across which the point spread function (PSF) varies greatly. MCAO
has the potential to provide, at least in the important NIR wavelength range from 1 to 2.5 micro-meters, quite uniform, near diffraction-limited, images [2].

The types of telescopes are divided as follows:

1- Refractors (Dioptric).

Refracting telescope: A thin lens has less spherical aberration than a thick one. Even after correcting for chromatic aberration has made the two elements of the objective considerably thicker, spherical aberration is still fairly low. Making the objective a meniscus lens minimizes it, because then the overall shape of the lens follows the curve of the surface where the incoming rays of light would be bent into their new desired direction while retaining a uniform spacing between them [3].

Special types of refracting telescope:

Achromatic: telescope has been color-corrected with the use of multiple lenses as shown in Fig (1-1).

Apochromatic: corrected for both chromatic and spherical aberration.

![Fig.(1-1). Refracting telescope [3].](image-url)
2- Reflectors (Catoptric).

Special types of reflector telescopes: Newtonian, Herschelian, Cassegrain, Dobsonian

Newton telescope: Is made up of a primary mirror, that gather the light and forms the light cone, then the secondary flat mirror, turns aside, this light cone, to an off-axis *focal plane*. To sustain the optical system we need a tube. The primary mirror is joined to the tube by the primary cell, and the secondary needs a holder that supports it into the tube. The secondary holder is supported to the tube by two, three or four arms. The global piece, holder and arms are called spider [4].

On the other hand, the eyepiece needs a focuser to support the eyepiece into the tube and thus be able to focus as in Fig (1-2).

*Fig.(1-2). Newton's telescope [4].*
3- Catadioptric.

Maksutov-Cassegrain telescope: A very expensive and high quality kind of telescope. In this telescope, the primary mirror is left as spherical. A thick glass element at the front of the telescope, with the same curvature on the front and back, acts as a corrector for the spherical aberration of the mirror. It also has a circular spot in the center that is coated on the inside to be a mirror; this mirror reflects the light that would normally be brought to a focus shortly beyond it, and, because of its curvature, delays the focusing of the reflected light until it goes out the back of the telescope through a hole in the center of the primary mirror [3].

![Diagram of Maksutov-Cassegrain telescope](image1.png)

*Fig.(1-3). Maksutov-Cassegrain telescope [3].*

Schmidt-Cassegrain telescope: Here, instead of a thick piece of glass with two spherical surfaces, correction is provided by a very thin piece of glass, flat on one side, and with an aspheric surface on the other.

![Diagram of Schmidt-Cassegrain telescope](image2.png)

*Fig.(1-4). Schmidt-Cassegrain telescope [3].*
1-2 Spider Definition

The spider is one of the key components in the optical setup. It holds the secondary mirror and allows adjustment of it. Adjustments can occur to the secondary mirror in both moving towards and away from the primary mirror, as well as the tilt of the secondary. It is held to the side of the tube by vanes. The vanes mount to the hub and the hub hold the mirror with room for adjustments. The vanes mount to the side of the tube.

The spider is an obstruction on the light path, thus, each obstruction has an edge, and due the wave nature of light, interference is produced. A way of interference is called diffraction. The diffraction produced by the spider causes the spikes that can be seen around bright objects. Each arm of the spider causes two spikes, and overlapping spikes will reinforce one over another. Depending on the number of arms, their positions, we will see a different pattern of the diffraction spikes [4].

Fig.(1-5). Some types of spiders and there diffraction spikes [4].
There are many ways to make a spider; here are some requirements to keep in mind:

- It should support the diagonal mirror firmly and without vibrations.
- It should allow the mirror to be tilted and positioned precisely in 3 dimensions, to allow exact collimation of the optical path (the mirror may also be rotated along the tube's optical axis, but this is not required).
- It should be designed not to give worse diffraction effects than necessary.

1-3 Literatures survey

The quantitative study of spider apertures and their effects on resolution has not taken a considerable attention. Therefore, our aim is to simulate various types of spider apertures and extend our study to include the effect of their parameters on the diffraction from these apertures.

Diffraction from secondary mirror spiders can significantly affect the image quality of optical telescopes; however, these effects vary drastically with the chosen image-quality criterion [5].

The qualitative effect of secondary-mirror spiders on the image-intensity distribution or point-spread function (PSF) of an optical telescope is well known to every amateur astronomer who has observed the familiar diffraction spikes accompanying star images. These effects can be important in certain scientific applications, depending on the image-quality criterion. However, more attention seems to have been devoted to this problem by amateur astronomers than by scientists and engineers. Several discussions concerning the use of curved spiders to reduce or eliminate these objectionable diffraction spikes have been
reported by amateur astronomers [6-10]. These include a diffraction less mount that is achieved by merely attaching the secondary mirror to a section of pipe that is attached to the telescope structure.

Astronomers also reported on an antidiffraction mask for a telescope that effectively eliminated the diffraction spikes at a considerable reduction in collecting area [11].

Diffraction effects that can degrade the image still exist; however, the azimuthal variations in the image-intensity distribution have been eliminated. Whether this results in a superior image depends on the application and the appropriate image-quality criterion for that application. Only few references have been found in the professional technical literature that deals specifically with spider diffraction. Several additional papers were found dealing with the more general subject of aperture configurations but include a peripheral discussion of spider diffraction effects [13].

The circular aperture of telescopes creates a sub-optimal diffraction pattern, the so-called Airy Pattern which is azimuthally symmetric. Currently the best way to diminish the Airy pattern is to use a coronagraph by using the combination of a stop in the focal plane that rejects a majority of the central bright object's light and a Lyot stop in the pupil plane to reject high frequency light [13-15]. Several recent ideas explore the use of alternative "apodized" apertures for high contrast imaging in the optical or near-infrared [16,17]. These designs revisit concepts first experimented with in the field of optics. Other designs, such as the band limited mask, seek to null the light from a central star in much the same way that a nulling interferometer performs [18].
All of these designs in theory can reach the contrast ratio necessary for imaging a planetary companion; however most of these designs have yet to be tested in the laboratory or on a real telescope where other concerns arise. The specific advantages of each idea cannot be determined until they are actually built or modeled in such a way as to simulate real engineering problems.

Adaptive optics (AO) systems remove the wave front distortion introduced by the Earth's atmosphere (or other turbulent medium) by means of an optical component(s) introducing a controllable counterwavefront distortion which both spatially and temporally follows that of the atmosphere. Telescope adaptive secondary mirrors (ASMs), which use an existing optical surface (the secondary mirror) as the wave front corrector, were proposed by Beckers (1989) to improve the performance of an adaptive optics system by elimination of the extra optical surfaces found in conventional AO systems.

It has been known that, if curved legs rather than the usual straight ones are used in the spider that supports the secondary optics in certain telescopes, the visible diffraction effect is reduced. Fraunhofer theory is used to calculate the diffraction effects due to the curved leg spider. Calculated and photographic diffraction patterns are compared for straight and curved leg spiders [19].

Diagrams are given of the isophotes (lines of equal light intensity) in two selected special cases and are compared with the corresponding diagram for an unobstructed aperture. It appears that when the central obstruction is large the bright central nucleus of the three-dimensional image becomes longer and narrower, so that focal depth and resolving
power are both increased. A central obstruction of ratio 0.25, on the other hand, is found to have practically no effect on the size and shape of the bright nucleus [20].

Traditional coronagraphy design is reviewed and extended to the case of arbitrary pupil geometries with adaptive optics (AO). By selecting a desired focal plane occulting mask and specifying the expected Strehl ratio achieved by the AO system, the design for a matched Lyot stop is given. The outline of the optimum Lyot stop is derived using general considerations of the residual halo from an on-axis source, as well as by maximizing the signal-to-noise ratio. A continuous transmission variant of the optimum Lyot stop is also found by means of a heuristic argument. It is suggested that essentially the same result may be found using a matched-filter analog from signal processing theory. The resulting Lyot stops for multi-segment telescope apertures can be quite unexpected, and a case with six circular segments is considered as an example [21].

Imaging extra solar planets has shown that high-contrast imaging can be achieved using specially shaped pupil masks [19].

Following successful implementation of tip/tilt secondary mirrors, most recent large telescope projects have considered the possibility of incorporating ASMs. This paper briefly reviews the development of ASMs and examines the issues which have arisen and also presents the predicted performance of an ASM system. It is concluded that adaptive secondary approach is an equally satisfactory or preferred solution to conventional AO systems [22].

An alternative to classical pupil apodization techniques (use of an amplitude pupil mask) is proposed by [23]. It is shown that an achromatic apodized pupil suitable for imaging of extrasolar planets can be obtained by reflection of an unapodized flat wavefront on two mirrors. By
carefully choosing the shape of these two mirrors, it is possible to obtain a contrast better than $10^{-9}$. Because this technique preserves both the angular resolution and light gathering capabilities of the unapodized pupil, it allows efficient detection of terrestrial extrasolar planets with a 1.5m telescope in the visible [23].

1-4 Thesis Organization.

The thesis is organized to study the parameters that are associated with central obstruction and spider vanes that hold the secondary mirror. The work is presented as follows:

1- Chapter one describes the general introduction on spider apertures and literature survey.
2- The image formation model is presented in chapter two.
3- Two-dimensional computer simulations of central obstruction and various spider vanes and their results are described in chapter three.
4- Chapter four contains the main conclusions and the future work.
Chapter Two
Image Formation Model

2-1 Introduction

Ideal image formation can be modeled as the convolution of the true brightness distribution of the object with the telescope point spread function. The PSF is the spatial frequency impulse response of the telescope, which is to say, the image of an ideal point source such as a star. An approximate model of the PSF can be computed as the Fourier transform of the field in the exit pupil of the telescope. For unaberrated, unobscured telescopes, this pupil function consists of a spherical wave front centered on the detector, with uniform intensity within the geometrically-defined circular aperture. The resulting PSF has the well-known form of the Airy pattern.

Ideal telescopes have PSFs that differ from the ideal Airy pattern. Optical aberrations due to misalignments, thermal gradients, out gassing, manufacturing errors and other effects change the configuration of the optics, causing the phase of the wave front propagated through the telescope to deviate from a perfect sphere. This will decrease the amplitude and increased width of the PSF, causing blurring of the image and decreasing the sensitivity of the instrument. Vignetting or shadowing incurred at stops, spatial filters, and obscurations such as mirror support pads, spiders and secondary mirrors, further decreases the PSF amplitude and can alter the halo of the PSF, adding diffraction rings and tendrils. These effects are very apparent in images taken with the first set of HST instruments.
For most telescopes, wave front phase and vignetting effects vary as a function of field angle (or focus setting). These induced aberrations cause the PSF to vary both in structure and amplitude from point to point over the detector. Again, HST images show the effects of strong spatial variance of the PSF. Detector characteristics are also significant. These include pixel size as well as non-ideal behaviors such as spatially-variable quantum efficiency or pixel cross-talk. Other effects, such as atmospheric turbulence can cause smearing of the PSF.

Image restoration can compensate for telescope aberrations, induced aberrations and detector effects by deconvolving a "known" PSF from the image data. Known jitter and turbulence effects can be compensated similarly. The variation of the PSF over the field can be handled by using multiple PSFs, each valid within a sub-region of the detector. The result of restoration using accurate PSFs is a recovered image whose accuracy is limited only by the noise content of the original image. These noise effects can be reduced by taking several images of the same source and averaging.

The effect of secondary mirror spiders on the image quality of an optical telescope for star images accompanying diffraction spikes as shown in Figs. (2-1a) and (2-1b). Curved spider configurations similar to those shown in Fig. (2-1c) that do not produce prominent diffraction spikes.
Fig. (2-1). Diffraction effects of secondary mirror spiders on telescope image quality [5].

2-2 Optical imaging system.

The fundamental equation to be used for the formation of an image by any ideal optical system is given by [25]:

\[
g(x, y) = \int \int f(x', y') \text{psf}(x - x', y - y') dx' dy'
\]

\[
= f(x, y) \otimes \text{psf}(x, y)
\] 

(2-1)

Where \( \otimes \) denotes convolution, \( g(x,y) \) is the observed image intensity, \( f(x,y) \) is the object intensity (i.e. what the object is really looks like), \( \text{psf}(x,y) \) is the point spread function and represents the image smearing. The Fourier transform of this equation becomes,
\[ G(u,v) = F(u,v)T(u,v) \quad \text{............... (2-2)} \]

Where \( G(u,v) \) and \( F(u,v) \) are, respectively, the complex Fourier transforms of the image intensity \( g(x,y) \) and the object intensity \( f(x,y) \); \( T(u,v) \), the transform of the psf, is an important function known as the optical transfer function (OTF). The modulus or amplitude of the complex function \( T(u,v) \) is called the modulation transfer function (MTF).

Most optical systems are expected to perform a predetermined level of image integrity. Photographic optics, photolithographic optics, contact lenses, video systems, fax and copy optics, and compact disk lenses only sample the list of such optical systems. A convenient measure of this quality level is the ability of the optical system to transfer various levels of detail from object to image. Performance is measured in terms of contrast (degree of gray) or modulation, and is related to the degradation of the image of a perfect source produced by the lens.

The MTF described the image structure as a function of its spatial frequencies, most commonly produced by Fourier transforming the image spatial distribution or spread function. Therefore, the MTF provides simple presentation of image structure information similar in form and interpretation to audio frequency response. The various frequency components can be isolated for specific evaluation.

OTF describes the response of the optical systems to known sources and is composed of two components: the MTF and Phase Transfer Function.

When an optical system produces an image using perfectly incoherent light, then the function, which describes the intensity in the image plane produced by a point object plane, is called the impulse response function. The input (object intensity pattern) and output (image intensity pattern) are related by the simple convolution, see eq.(2-1).
Since it is impossible to construct a truly infinitesimal source, one cannot directly measure the response of a system to a point impulse. The impulse response and hence the transfer function must be derived rather than generated.

MTF can be specified either at a single wavelength or over a range of wavelengths, depending upon the application. MTF allows full spectrum specification and testing.

It is important to study the effect of telescope aperture on the actual point spread function and the modulus transfer function.

The pupil function, $H(\eta, \gamma)$, in the absence of atmospheric turbulence is given by the following equation [23]:

$$
H(\eta, \gamma) = \begin{cases} 
1 & \text{if} \left( (\eta - \eta_c)^2 + (\gamma - \gamma_c)^2 \right)^{1/2} < R \\
0 & \text{otherwise} 
\end{cases} \quad \ldots \ldots (2-3)
$$

where $R$ is the radius of a telescope aperture and $\gamma_c, \eta_c$ are the center of a 2-Dimensional aperture.

The instantaneous PSF of the telescope system alone is given by:

$$
psf(x, y) = |FT[H(\eta, \gamma)]|^2 \quad \ldots \ldots (2-4)
$$

where FT denotes Fourier transform operator.

The MTF of a system may be measured with an interferometer by one of two methods: auto-correlating the pupil function of the lens-under-test or analyzing the PSF that calculated by Fourier transforming the pupil wave front. This is very convenient for systems which are suitable for testing in an interferometer and do not exhibit significant chromatic aberrations, and whose wave front errors do not vary substantially over
the wavelength of interest [25]. The transfer function of an incoherent optical system is equal to the autocorrelation of the pupil function [26], the optical transfer function (OTF), $T(u, v)$, could be written in terms of telescope aperture, $H(\eta, \gamma)$, as,

\[
T(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H(\eta', \gamma') H^*(\eta + \eta', \gamma + \gamma')\, d\eta\, d\gamma' \quad \cdots \cdots \quad (2-5)
\]

The variables $(\eta, \gamma)$ represent distance in the pupil, and are related to the spatial frequency variables $(u, v)$ by:

\[
\eta = f \lambda u; \quad \gamma = f \lambda v \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cd DOT
(or contrast) in the image as a function of spatial frequency. It has a value of unity at the origin (zero spatial frequency) and decreases to zero at the cutoff spatial frequency. The limiting resolution is defined as the reciprocal of this cutoff spatial frequency.

From the autocorrelation theorem or Fourier transform theory, the OTF and the PSF are Fourier transforms of each other [29]. These relationships are illustrated schematically in Fig.(2-2), along with a variety of commonly used image-quality criteria.

The MTF has gained almost universal acceptance as the image-quality criterion of choice for a wide range of imaging applications involving diffraction-limited optics, or for those suffering from image degradation caused by aberrations. Further image degradation caused by image motion has also been modeled by an appropriate transfer function for image jitter and drift [30, 31].

\[ H(x,y) \xrightarrow{\text{\star}} OTF(\xi,\eta) \xrightarrow{\text{MTF}} |OTF| \]

\[ A(\xi,\eta) \xrightarrow{\text{\sqrt{\frac{2}{\pi}}}} PSF(x,y) \]

*Fig. (2-2). Relationship between the complex pupil function, OTF, and the PSF [25].*
2-3 Imaging with turbulent atmosphere

The turbulent phenomena associated with heat flow and winds in the atmosphere, causing the density of air fluctuating, in space and time, occur predominantly due to the winds at various highest, convection in and around the building and the dome, any obstructed location near the ground, off the surface of the telescope structure, inside the primary mirror cell,…etc. The cells of differing sizes and refractive indices produced by this phenomena, move rapidly across the path light, causing the distortion on the shape of the wave front and variations on the intensity and phase.

Large scale temperature inhomogeneities caused by differential heating of different portions of the Earth's surface produce random micro-structures in the spatial distribution of temperature which, in turn, cause the random fluctuations in the refractive index of air. These large scale refractive index inhomogeneities are broken up by turbulent wind and convection, spreading the scale of the inhomogeneities to smaller sizes [23].

In the case where we are imaging through a random medium, the pupil function may be split into a product of two functions, one representing the effect of the random medium, and one representing the pupil function of the imaging component;

\[ H(\eta, \gamma) = A(\eta, \gamma)Z(\eta, \gamma) \quad \ldots \ldots \ldots \quad (2-8) \]

Where \( A(\eta, \gamma) \) is the pupil function of the imaging part of the system which is exactly the same as that given by eq.(2-3) and \( Z(\eta, \gamma) \) is the complex amplitude at the imaging aperture due to a point source in the object plane.
In the case under condition, $Z(\eta, \gamma)$ is a random variable. Substituting eq.(2-8) into (2-5) yields,

$$T(u, v) = \int \int A(\eta', \gamma') A^* (\eta + \eta', \gamma + \gamma') Z(\eta', \gamma') Z^* (\eta + \eta', \gamma + \gamma') d\eta' d\gamma'$$

...................... (2-9)

The transfer function of the random medium / imaging system for normal time-averaged imaging may be found by taking the ensemble average of $T(u, v)$. Thus,

$$\langle T(u, v) \rangle = \int \int A(\eta', \gamma') A^* (\eta + \eta', \gamma + \gamma') \langle Z(\eta', \gamma') Z^* (\eta + \eta', \gamma + \gamma') \rangle d\eta' d\gamma'$$

...................... (2-10)

The autocorrelation function of $Z(\eta, \gamma)$ is defined as:

$$C_Z (\eta, \gamma) = \langle Z(\eta', \gamma') Z^* (\eta + \eta', \gamma + \gamma') \rangle \quad \ldots\ldots (2-11)$$

The transfer function of the optical system only may be written as:

$$T_o (u, v) = \int \int A(\eta', \gamma') A^* (\eta + \eta', \gamma + \gamma') d\eta' d\gamma' \quad \ldots\ldots (2-12)$$

and therefore,

$$\langle T(u, v) \rangle = T_o (u, v) C_Z (\lambda f u, \lambda f v) \quad \ldots\ldots (2-13)$$
The form of $C_{\lambda}(\Delta f u, \Delta f v)$ falls to zero much faster than $T_{\lambda}(u, v)$, and the resolution is therefore limited by the seeing [26].

2-4 Image Quality Criteria

If one is trying to observe bright binary stars visually, the curved spiders that eliminate diffraction spikes are probably desirable. However, the diffraction spikes caused by conventional (narrow) spiders do not significantly broaden the image core. The full width at half-maximum (FWHM) of the PSF, an appropriate image-quality criterion when bright point sources are observed, is therefore not degraded by diffraction effects from secondary mirror spiders [5].

The complex pupil function describes the amplitude (aperture shape including obstructions and spider configuration) and phase (wave-front aberrations) variations in the exit pupil of the telescope that determine image quality. Wave-front aberrations (which are neglected in this discussion of diffraction effects) are rendered observable and measured by interferometric techniques. Single-number merit functions derivable from interferometric data include the rms wave-front error and the peak-to-valley wave front error.

The PSF is the squared modulus of the Fourier transform of the complex pupil function as illustrated in Fig. (2-2). The intermediate quantity called the amplitude spread function is not an observable quantity with ordinary sensors. Frequently used single number merit functions (or image-quality criteria) obtained from the PSF are the resolution (FWHM), the Strehl ratio, and the fractional encircled energy. Fractional encircled energy, or the closely related half-power radius, of the PSF have become common image-quality requirements imposed on telescope manufacturers in recent years. These image-quality criteria are
particularly relevant if the telescope is to be used to collect light and place the image on the slit of a spectrographic instrument.

The autocorrelation theorem of Fourier-transform theory permits us to define the OTF as the normalized autocorrelation of the complex pupil function. Various properties of the OTF, or its modulus, the modulation transfer function, may provide more appropriate image-quality criteria if the application involves studying fine detail in extended objects. Limiting resolution and the transfer factor at a specific spatial frequency are single-number merit functions derivable from the OTF.

We deal quantitatively with diffraction effects of secondary mirror spiders on several different image-quality criteria including the Strehl ratio, the fractional encircled energy, and the modulation transfer function.

2-4-1 Strehl Ratio

The Strehl ratio, defined as the ratio of the peak irradiance of an aberrated PSF to the peak irradiance of the diffraction-limited PSF, is a commonly used image-quality criterion. A slight modification of this definition (the diffraction-limited peak irradiance with spiders divided by the diffraction-limited peak irradiance without spiders) is appropriate for this study.

From the central ordinate theorem of the Fourier transform theory (which states that the area of a function is equal to the central ordinate of its Fourier transform) [33] and the relationships shown in Fig. (2-2), it is clear that the Strehl ratio can be expressed as

$$Strehl\_ratio \equiv S = \left(\frac{A_{annulus} - A_{spider}}{A_{annulus}}\right)^2 \quad \text{....... (2-14)}$$
Where $A_{annulus}$ is the area of the annular aperture without spiders and $A_{spider}$ is the total area of all spiders.

### 2-4-2 Fractional Encircled Energy

The complex amplitude distribution in the focal plane of a telescope is given by the product of some complex exponentials with the Fourier transform of the complex pupil function evaluated at spatial frequencies $\xi = x_2 / \lambda f$ and $\eta = y_2 / \lambda f$ [27].

$$U(x_2, y_2) = \frac{\exp(ikf)}{i\lambda f} \exp\left[\frac{i\pi(x_2^2 + y_2^2)}{\lambda f}\right] F\{U_1(x_1, y_1)\} |_{\xi=x_2/\lambda f, \eta=y_2/\lambda f}$$

………… (2-15)

Here $k = (2\pi) / \lambda$, $f$ is the focal length of the telescope, $F$ denotes the Fourier-transform operation, and the complex pupil function is given by

$$U_1(x_1, y_1) = B(x_1, y_1)T_1(x_1 / D, y_1 / D) \exp[ikW(x_1, y_1)] \quad ...\quad (2-16)$$

Where $B(x_1, y_1)$ is the incident amplitude distribution (field strength), $T_1(x_1 / D, y_1 / D)$ is the aperture function of outer diameter $D$ (including any obscurations of spiders), and $W(x_1, y_1)$ is the wave-front aberration function [34] describing any phase variations in the exit pupil of the telescope.

For a uniform amplitude, normally incident plane wave (no aberrations), the pupil function is just the constant $B$ times the aperture function,
and the irradiance distribution in the image plane illustrated in Fig.(2-3) is given by

\[
I(x_2, y_2) = |U(x_2, y_2)|^2 = \frac{B^2}{\lambda^2 f^2} \left| F \left[ \frac{T_1(x_1, y_1)}{D} \right] \right|^2 \quad \text{........ (2-18)}
\]

Fig.(2-3). Diffraction-limited irradiance distribution in the focal plan of a telescope depending on the dimensions of the pupil function in the exit pupil, the focal length of the telescope, the wavelength, and the incident field strength [5].

By applying the central ordinate theorem of the Fourier transform theory to eq. (2-18), we see that the on-axis irradiance in the image plane is given by [5]
\[ I(0,0) = \frac{B^2}{\lambda^2 f^2 A_{\text{aperture}}^2} \quad \text{............. (2-19)} \]

The normalized irradiance distribution (normalized to unity at the origin) is thus expressed in dimensionless coordinates \( x = x_2 D / \lambda f \) and \( y = y_2 D / \lambda f \) as

\[
I_n(x, y) = \frac{I(x_2, y_2)}{I(0,0)} = \frac{1}{A_{\text{aperture}}^2} \left| F \left\{ T_1 \left( \frac{x_1}{d}, \frac{y_1}{D} \right) \right\}_{\xi = x_2 \lambda f, \eta = y_2 \lambda f} \right|^2 \quad \text{............... (2-20)}
\]

Where \( A_{\text{aperture}} \) is just the area of the aperture. The fractional encircled energy, a commonly used image quality criterion, is defined as the radiant energy contained in a circle of radius \( r_2 \) divided by the total radiant energy reaching the focal plane:

\[
EE(r_2) = \frac{\int_0^{2\pi} \int_0^\pi I(x_2, y_2) r_2 dr_2 d\phi}{\int_\infty^{-\infty} \int_\infty^{-\infty} I(x_2, y_2) dx_2 dy_2} \quad \text{........... (2-21)}
\]

Substituting eq. (2-18), we see that the denominator of eq. (2-21) can be written as
\[
\int_0^\infty \int_0^\infty I(x_2, y_2) dx_2 dy_2 = \frac{B^2}{\lambda^2} \int_0^\infty \int_0^\infty \left| T_1 \left( \frac{x_1}{D}, \frac{y_1}{D} \right) \right|^2 \lambda^2 f^2 d\xi d\eta
\]

\[\text{(2-22)}\]

and when Rayleigh’s theorem is applied [33,35], which states that the integral of the squared modulus of a function is equal to the integral of the squared modulus of its Fourier transform (which corresponds to Parseval’s theorem for the Fourier series), Eq. (2-22) is equal to

\[
\int_0^\infty \int_0^\infty I(x_2, y_2) dx_2 dy_2 = B^2 \int_0^\infty \int_0^\infty \left| T_1 \left( \frac{x_1}{D}, \frac{y_1}{D} \right) \right|^2 dx_1 dy_1
\]

\[\text{\text{(2.23)}}\]

But for a binary amplitude aperture that has a transmittance of either unity or zero,

\[
\left| T_1 \left( \frac{x_1}{D}, \frac{y_1}{D} \right) \right|^2 = T_1 \left( \frac{x_1}{D}, \frac{y_1}{D} \right)
\]

\[\text{\text{(2-24)}}\]

Hence the denominator of eq. (2-16) can be written as

\[
\int_0^\infty \int_0^\infty I(x_2, y_2) dx_2 dy_2 = B^2 A_{\text{aperture}}
\]

\[\text{\text{(2-25)}}\]

Now, substituting
\[ I(x_2, y_2) = \frac{B^2 A_{\text{aperture}}^2}{\lambda^2 f^2} I_n(x, y) \quad \ldots \ldots \quad (2-26) \]

Into the numerator of eq. (2-21) and noting that \( r = \frac{r_2 D}{\lambda f} \) and \( dr = \frac{dr_2 D}{\lambda f} \) in dimensionless coordinates, we obtain the fractional encircled energy in terms of the normalized irradiance distribution:

\[ EE(r) = \frac{A_{\text{aperture}}}{D^2} \int_{0}^{2\pi} \int_{0}^{\pi} I_n(x, y) r dr d\phi \quad \ldots \ldots \quad (2-27) \]
Chapter Three
Modeling and Simulation

3-1 Imaging with normal aperture

Consider an extremely distant quasimonochromatic point source located on the optical axis of a simple imaging system. In the absence of atmospheric turbulence, this source would generate a plane wave normally incident on the lens. The instantaneous PSF of the entire telescope system is given by Eq.(2-4).

\[ H(\eta, \gamma) \]

as given by Eq.(2-3) is taken to be a two dimensional circular function. The actual size of this array (M by N) is taken to be 512 by 512 pixels. This size is taken as large as possible in order to keep the theoretical diffraction limit of the telescope of interest to be truncated at zero value inside this array.

The Modulation transfer function, MTF is given by:

\[ MTF(u, v) = \left| FT[psf(x, y)] \right| \quad \ldots \ldots \quad (3-1) \]

or it may be written by taking the absolute of Eq.(2-5), i.e.

\[ MTF(u, v) = \left| T(u, v) \right| = \left| \int \int H(\eta', \gamma')H^*(\eta + \eta', \gamma + \gamma')d\eta'd\gamma' \right| \quad \ldots \ldots \quad (3-2) \]

The PSF and MTF for uniform telescope apertures at different radius (R) are shown in Fig.(3-1) and (3-2).
Fig.(3-1). PSF and MTF as a function of telescope diameter (R=100 pixels).

It should be pointed out here that the central pixel of Fig.(3-1d) represent the central spike of the PSF, and the central pixel of Fig.(3-1e) represent the highest frequency components value.
These two figures demonstrate that the faint details of PSF for R=100 are much sharper than that of R=25 pixels.

The PSF and MTF for annular aperture of R=100 pixels and secondary obstruction =25 pixels are shown in Fig.(3-3) and (3-4).
In the presence of central obstruction, the following parameters must be taken into account.

\[ r = \text{radius of the obstructing circle.} \]

\[ \epsilon = \frac{r}{R}, \text{ ratio of the radius of obstructing circle to the radius of primary aperture.} \]

Fig. (3-3). PSF and MTF for telescope annular aperture, \( \epsilon = 0.25 \).
The central lines through the MTF of uniform aperture [Fig.(3-1e)] and through the MTF of annular aperture Fig(3-3e) is shown in Fig(3-4).

![Fig.(3-4). Central plots through a- Fig.(3-1e) and b- Fig.(3-3e).](image)

It is clear from this figure that the low frequency components of annual aperture is clearly lower than that of uniform aperture, but on the other hand, the high frequency components are slightly higher in the case of annular aperture.

It is so important to find a standard or theoretical data that one could normalize these values. If we say that the delta function is a spike of infinite height, zeros width and unit area. More precisely, we may specify it as the limit of a function. This function is implemented in mathematics as a Dirac's delta function.

\[
\delta(x) = \lim_{W \to 0} \left( \frac{1}{x\pi} \sin(x/W) \right) \quad \ldots \ldots \quad (3-3)
\]

\[
\delta(x) = \lim_{W \to 0} \left( \text{Exp}(-x^2/W^2) \right) \quad \ldots \ldots \quad (3-4)
\]

The minimum possible value of \(W\) must verify the following fundamental delta function property, i.e.,
\[
O(x, y) \otimes \delta(x, y) = O(x, y) \quad \ldots \ldots \quad (3-5)
\]

To achieve this property, \( \delta(x, y) \) is taken to be an array of size \( M=512 \) pixels by \( N=512 \) pixels with the central pixel value of unity magnitude (i.e., all the values are zeros except the central pixel value = 1), i.e.,

\[
\sum_{y=1}^{M} \sum_{x=1}^{N} \delta(x, y) = 1 \quad \ldots \ldots \quad (3-6)
\]

\[
\frac{1}{MN} \sum_{y=1}^{M} \sum_{x=1}^{N} \delta(x, y) = 7.97 \times 10^{-7} \quad \ldots \ldots \quad (3-7)
\]

\[
\frac{1}{MN} \sum_{u=1}^{M} \sum_{v=1}^{N} \text{MTF}(u, v) = \frac{1}{MN} \sum_{y=1}^{M} \sum_{x=1}^{N} \text{ABS}(FT[\delta(x, y)]) = 1 \quad \ldots \ldots \quad (3-8)
\]

It should be pointed out here that as \( R \to \infty \), the PSF becomes delta function and MTF becomes nearly constant.
3-2 Imaging with spiders.

In the presence of spiders, the telescope apertures are look like the followings:

**Type (A) – Line spiders,**

(A-1) - Line spiders with $\varepsilon = 0.25$, $\delta = \text{the width of vanes in pixels} = 3$.  

(A-2) - Line spiders with $\varepsilon = 0.25$, separation angle $= 120^\circ$, $\delta = 3$ pixels.
**Type (B) – Curved spiders.**

**Simulation of curved spider:**

\[ \theta = \text{arc in degree} \]
\[ w = \text{width of spider} \]
\[ X1 = Ra - Rs \]

where \( Ra = \text{Radius of primary mirror.} \)
\[ Rs = \text{Radius of secondary mirror.} \]

\[ z = \frac{\theta}{2}, \text{in radian} \]

1 - \[ I - R = \frac{0.5X1}{\sin(z)}, \text{R is the radius of the circle that used to extract the spider.} \]

\[ p1 = R \cos \theta \]
\[ D = R - p1 \]

2 - The extraction is starting from the point \( P \left( c + p1, c - \frac{X1}{2} \right) \), for an area of width \( R - p1 + w \) and length of \( X1 \).

3 - This area could be rotated as desired to get the shape that we want for spider generation.
(B-1) Curved spider, $\varepsilon = 0.25$, $\delta = 3$ pixels, vane curve $= 60^\circ$, separation angle $= 90^\circ$. 

Fig.(3-5). Simulation of curved spider (Vane).
(B-2)- Curved spider, \( \varepsilon = 0.25 \), \( \delta = 3 \) pixels, vane curve = \( 90^\circ \), separation angle = \( 90^\circ \).
(B-3)- Curved spider, $\varepsilon = 0.25$, $\delta = 3$ pixels, vane curve = 60° and 90°, separation angle = 120°.

These kind of apertures are all simulated in order to study the effects that introduced by the secondary mirror and the spiders on the actual diffraction. The PSF and MTF for distance point source are computed via equations (2-3), (3-1), and (3-2) respectively.
The results of using spiders of type (A-1) with $\varepsilon = 0.25$ and $\delta = 3$ are shown in figs. (3-6), (3-7) and (3-8).

(a) One vane.
(b) Two vanes.
(c) Three vanes.
(d) Four vanes.
The central portions (15 by 15 pixels) of the PSFs for the four apertures are then extracted and plotted in Fig.(3-7).

*Fig.(3-7). Extracted versions of the PSFs of Fig.(3-6). (a), (b), (c) and (d) corresponding to Fig.(3-6).*
The MTFs of these four PSFs are also computed and plotted in Fig.(3-8).

Fig.(3-8). MTFs for type (A-1), $\varepsilon = 0.25$ and $\delta = 3$ Pixels.
   
   a- One vane.
   b- Two vanes.
   c- Three vanes.
   d- Four vanes.
The results of using spiders of type (A-1) with $\epsilon = 0.25$ and $\delta = 25$ pixels are shown in Figs. (3-9), (3-10) and (3-11).

Fig. (3-9). Telescope apertures and the Log(1+psf) for type (A-1), $\epsilon = 0.25$ and $\delta = 25$ pixels.

- **a**- One vane.
- **b**- Two vanes.
- **c**- Three vanes.
- **d**- Four vanes.
The central portions (15 by 15 pixels) of the PSFs for the four apertures are then extracted and plotted in Fig.(3-10).

Fig.(3-10). Extracted versions of the PSFs of Fig.(3-9). (a), (b), (c) and (d) corresponding to Fig.(3-9).
The MTFs of these four PSFs are also computed and plotted in Fig.(3-11).

Fig.(3-11). MTFs for type (A-1), \( \epsilon = 0.25 \) and \( \delta = 25 \) pixels.

\( a- \) One vane.
\( b- \) Two vanes.
\( c- \) Three vanes.
\( d- \) Four vanes.

The central line of the MTF of a uniform aperture and of Figs.(3-11a, b, c & d) are shown in Fig.(3-12).

Fig.(3-12). Central plots through, Figs.(3-1e, 3-8a, 3-8b, 3-8c, 3-8d) are shown in a, b, c, d, and e respectively.
The corresponding plot of Fig.(3-12) with $\delta = 25$ pixels is shown in Fig.(3-13).

![Central plots through, Figs.(3-1e, 3-11a, 3-11b, 3-11c, 3-11d) are shown in a, b, c, d, and e respectively.](image)

It should be pointed out here that imaging with central obstructions and spiders will result in reducing the low frequency components and slightly boosting the high frequency components. This will slightly lead to increase the resolution.

Now, it is important to examine the extent of the PSF. This is done by normalizing the PSFs of that using all apertures as given in type A and B. The normalization is taken by reshaping the PSFs to one at maximum value and the average spatial intensity value is calculated following the equation,

$$\text{Average PSF} = \frac{1}{NM} \sum_{y=1}^{M} \sum_{x=1}^{N} \frac{\text{psf}(x, y)}{\text{psf}(0,0)} \quad \ldots \ldots \quad (3-9)$$

where $\text{psf}(0,0)$ is the central value of the $\text{psf}(x, y)$.

Fig. (3-14) shows the average PSF for $\epsilon = 0.25$ and $\delta$ is ranging from zero to 25 pixels using 1, 2, 3, and 4 vanes.
Fig.(3-14). Average PSF as a function of $\delta$ for type (A-1), ($\epsilon = 0.25$).

The average PSF for the normal aperture of (R=100 pixels) without any obstruction and vanes is $3.183 \times 10^{-5}$ as in Fig (3-1), and the average PSF for the annular aperture of $\epsilon = 0.25$ is $3.3949 \times 10^{-5}$ as in Fig (3-3).

Fig. (3-10) demonstrates the fact that with one spider, the effect of widening the size of the vane is not significant.

The corresponding average MTF are also normalized to one at maximum.

$$
Average_{-MTF} = \frac{1}{MN} \sum_{y=1}^{M} \sum_{x=1}^{N} \frac{MTF(x,y)}{MTF(0,0)} \quad \ldots \ldots \ (3-10)
$$

where $MTF(0,0)$ is the central value of the $MTF(x,y)$.

The results are then presented in Fig. (3-15).
Fig.(3-15). Average MTF as a function of $\delta$ for type (A-1), ($\epsilon = 0.25$).

The average MTF of the normal aperture (R=100 pixels) without any obstruction and spider is 0.1198 as in Fig (3-1), and the MTF of the annular aperture of $\epsilon = 0.25$ is 0.1124 as in Fig (3-3). The average MTF is not significantly reduced with one vane.

Fig. (3-16) shows the average PSF for $\epsilon = 0.5$ and $\delta$ is ranging from zero to 25 pixels using 1, 2, 3, and 4 vanes.

Fig.(3-16). Average PSF as a function of $\delta$ for type (A-1), ($\epsilon = 0.5$).
Fig. (3-17) illustrates the average MTF for $\epsilon = 0.5$ and the $\delta$ is ranging from zero to 25 pixels using 1, 2, 3, and 4 vanes.

![Average MTF Graph](image)

**Fig.(3-17). Average MTF as a function of $\delta$ for type (A-1), ($\epsilon = 0.5$).**

Fig. (3-18) demonstrates the average PSF for $\epsilon = 0.75$ and $\delta$ is ranging from zero to 25 pixels using 1, 2, 3, and 4 vanes.

![Average PSF Graph](image)

**Fig.(3-18). Average PSF as a function of $\delta$ for type (A-1), ($\epsilon = 0.75$).**
Fig. (3-19) describes the behavior of the average MTF for $\varepsilon = 0.75$ and $\delta$ is ranging from zero to 25 pixels using 1, 2, 3, and 4 vanes.

![Graph showing average MTF as a function of $\delta$ for different vanes.]

Fig.(3-19). Average MTF as a function of $\delta$ for type (A-1), ($\varepsilon = 0.75$).

The strehl ratios in terms of the PSF and MTF following eq.(2-14) are given by

\[
Strehl_{PSF} = \left( \frac{\sum \sum (PSF_{\text{uniform}} - PSF_{\text{annular}})}{\sum \sum PSF_{\text{uniform}}} \right)^2 \quad ........ (3-11)
\]

\[
Strehl_{MTF} = \left( \frac{\sum \sum (MTF_{\text{uniform}} - MTF_{\text{annular}})}{\sum \sum MTF_{\text{uniform}}} \right)^2 \quad ........ (3-12)
\]
Fig. (3-20) presents the Strehl ratio for $\varepsilon = 0.25$ and $\delta$ is ranging from one to 25 pixels using 1, 2, 3 and 4 vanes as function of average PSF.

![Graph showing Strehl ratio as a function of PSF for type (A-1), $\varepsilon = 0.25$, $\delta = 1, 3, 5, ..., 25$ pixels.]

Fig. (3-20) *Strehl ratio as a function of PSF for type (A-1), $\varepsilon = 0.25$, $\delta = 1, 3, 5, ..., 25$ pixels.*

Fig. (3-21) shows the Strehl ratio for $\varepsilon = 0.25$ and $\delta$ is ranging from one to 25 pixels using 1, 2, 3, and 4 vanes as function of average MTF.

![Graph showing Strehl ratio as a function of MTF for type (A-1), $\varepsilon = 0.25$, $\delta = 1, 3, 5, ..., 25$ pixels.]

Fig. (3-21) *Strehl ratio as a function of MTF for type (A-1), $\varepsilon = 0.25$, $\delta = 1, 3, 5, ..., 25$ pixels.*
The results of using spiders of type (A-2) with $\epsilon = 0.25$ and $\delta = 9$ pixels are shown in Figs. (3-22), (3-23) and (3-24).

![Telescope apertures and the Log(1+psf) for type A-2, $\epsilon = 0.25$ and $\delta = 9$ pixels.](image)

**Fig. (3-22).** Telescope apertures and the Log(1+psf) for type A-2, $\epsilon = 0.25$ and $\delta = 9$ pixels.

- **a-** Telescope aperture, $\epsilon = 0.25$ and $\delta = 9$.
- **b-** Log(1-psf). The aperture in (a) is used.
- **c-** Log(1+psf). A uniform aperture in the absence of obstruction and no vanes is used.
The central portions (15 by 15 pixels) of the PSFs for the two apertures are then extracted and plotted in Fig. (3-23).

Fig. (3-23). Extracted versions of the PSF for type (A-2), $\epsilon = 0.25$ and $\delta = 9$.

- a- Image PSF with central obstruction and three spiders (size 15 by 15 pixels).
- b- Image PSF of uniform aperture (without central obstruction and spiders).
- c- Surface plot of (a).
- d- Surface plot of (b).
The MTFs of these two PSFs are also computed and plotted in Fig.(3-24).

Fig.(3-24). MTFs for type(A-2), $\epsilon = 0.25$ and $\delta = 9$ pixels.

- **a-** MTF with central obstruction and three vanes.
- **b-** MTF that using uniform aperture (without obstruction and vane).
- **c-** Central plots through (a) and (b).
Fig. (3-25) shows the average PSF for $\varepsilon = 0.25$ and $\delta$ is ranging from one to 25 pixels using 3 vanes.

![Average PSF graph](image)

**Fig.(3-25). Average PSF as a function of $\delta$ for type (A-2), $\varepsilon = 0.25$.**

Fig. (3-26) illustrates the average MTF for $\varepsilon = 0.25$ and $\delta$ is ranging from one to 25 pixels using 3 vanes.

![Average MTF graph](image)

**Fig.(3-26). Average MTF as a function of $\delta$ for type (A-2), $\varepsilon = 0.25$.**
Fig. (3-27) shows the Strehl ratio for $\epsilon = 0.25$ and $\delta$ is ranging from one to 25 pixels using 3 vanes as function of average PSF.

![Graph showing Strehl ratio as a function of PSF](image)

**Fig. (3-27) Strehl ratio as a function of PSF for type (A-2), $\epsilon = 0.25$, $\delta = 1, 3, 5, ..., 25$ pixels.**

Fig. (3-28) describes the Strehl ratio for $\epsilon = 0.25$ and $\delta$ is ranging from one to 25 pixels using 3 vanes as function of average MTF.

![Graph showing Strehl ratio as a function of MTF](image)

**Fig. (3-28) Strehl ratio as a function of MTF for type (A-2), $\epsilon = 0.25$, $\delta = 1, 3, 5, ..., 25$ pixels.**
The results of using spiders of type (B-2) with $\varepsilon = 0.25$, $\delta = 5$ pixels and the curve of spider = $90^\circ$ are shown in Figs. (3-29), (3-30) and (3-31).

Fig.(3-29). Telescope apertures and the Log(1+psf) for type (B-2), $\delta = 5$ pixels, $\varepsilon = 0.25$.

a- One vane, b- Two vanes, c- Three vanes and d- Four vanes.
The central portions (15 by 15 pixels) of the PSFs for the four apertures are then extracted and plotted in Fig. (3-30).

Fig. (3-30). Extracted versions of the PSFs of Fig. (3-29). (a), (b), (c) and (d) corresponding to Fig. (3-29).
The MTFs of these four PSFs are also computed and plotted in Fig.(3-31).

Fig.(3-31). MTFs for type (B-2), $\delta = 5$ pixels, $\varepsilon = 0.25$.

- a- One vane.
- b- Two vanes.
- c- Three vanes.
- d- Four vanes.
Now, it is so important to study the effect of curved spider on the actual PSF and MTF.

Fig. (3-32) shows the average PSF for $\epsilon = 0.25$, type (B) of spiders with 1 curved spider, $\delta$ is ranging from one to 15 pixels using spider curve of 30, 45, 60, 75 and 90°.

**Fig (3-32).** Average PSF as a function of $\delta$ for type (B), $\epsilon = 0.25$ and one curved spider is used.

Fig. (3-33) presents the average MTF for $\epsilon = 0.25$, type (B-2) of spiders with 1 curved spider, $\delta$ is ranging from one to 15 pixels using spider curve of 30, 45, 60, 75 and 90°.
Fig (3-33). Average MTF as a function of $\delta$ for type (B), $\varepsilon = 0.25$ and one curved spider is used.
The results of using spiders of type (B-3) with $\varepsilon = 0.25$, $\delta = 3$ pixels and the curve of spider $= 45^\circ$ and $90^\circ$ are shown in Figs. (3-34), (3-35) and (3-36).

Fig. (3-34). Telescope apertures and the Log(1+psf) for type (B-3), $\delta = 3$ pixels, $\varepsilon = 0.25$.

a- Three vanes, vane curve is $45^\circ$.

b- Three vanes, vane curve is $90^\circ$. 
The central portions (15 by 15 pixels) of the PSFs for the two apertures are then extracted and plotted in Fig. (3-35).

Fig.(3-35). Extracted versions of the PSFs of Fig.(3-34). (a) and (b) corresponding to Fig.(3-34).

The MTFs of these two PSFs are also computed and plotted in Fig.(3-36).

Fig.(3-36). MTFs for type (B-3), $\delta = 3$ pixels, $\epsilon = 0.25$.

- Three vanes of curve 45 degree.
- Three vanes of curve 90 degree.
Now it is of interest to study the effect of spiders and central obstruction on the actual resolution of an image of a binary star,

The binary star that observed by a given telescope is simulated as follows:

1 – The PSF is generated as before using apertures of $e = 0, 0.25, 0.5$ and $0.75$.
2 – The PSF are shifted 2 pixels diagonally, i.e., 2 pixels in y-direction and 2 pixels in x-direction.
   The result is a shifted (2 by 2) version of step(1).
3 – Adding step(1) to step(2) resulting a binary star, $B(x,y)$, of a separation of 2 by 2 pixels.
   This separation is chosen in order to clearly qualify the results.

The autocorrelation function (AC) of an image of binary star, or equivalently its power spectrum (PS), are computed as:

$$PS(x, y) = \left| FT(B(x, y)) \right|^2$$  
$$AC(x, y) = \left| FT^{-1}(PS(x, y)) \right|$$

---------- (3-13)  

---------- (3-14)
The power spectrums of an image of binary star using type (A-1) of spiders, separation (2, 2), are then shown in Fig. (3-37).

Fig.(3-37). Images of binary stars and their Power spectra for type (A-1), separation (2,2), $\varepsilon = 0.25$, $\delta = 3$.

a- Image of binary star with central obstruction and 4 spiders.

b- Image of binary star of ordinary aperture (without obstruction and spider).

c- Power spectrum with central obstruction and 4 spider.

d- Power spectrum of ordinary aperture (without obstruction and spider).

e- Diagonal line through (c).

f- Diagonal line through (d).
Fig. (3-38) shows the average power spectrum for $\epsilon = 0.25$ and $\delta$ is ranging from zero to 25 pixels using 1, 2, 3, and 4 vanes for a binary star of separation (2,2).

![Graph showing average power spectrum as a function of $\delta$ for type (A-1), $\epsilon = 0.25$.](image)

Fig. (3-38). Average power spectrum as a function of $\delta$ for type (A-1), $\epsilon = 0.25$.

A quantitative quality measure that suggested in this study is called contrast power spectrum or relative power spectrum (RPS). This is to calculate the ratio of the peak power spectrum (D.C. Spike or maximum peak) to the secondary peak. This measure gives you a clear picture on the contrast of the high frequency components as shown in the following figure.
Fig.(3-39) shows the RPS for $\varepsilon = 0.25$ and $\delta$ is ranging from zero to 25 pixels using 1, 2, 3 and 4 vanes for a binary star with separation(2,2).

![RPS Graph](image1)

**Fig(3-39). RPS as a function of $\delta$ for type (A-1), $\varepsilon = 0.25$.**

Fig. (3-40) shows the average power spectrum for $\varepsilon = 0.5$ and $\delta$ is ranging from zero to 25 pixels using 1, 2, 3, and 4 vanes for a binary star of separation (2,2).

![Power Spectrum Graph](image2)

**Fig.(3-40). Average power spectrum as a function of $\delta$ for type (A-1), $\varepsilon = 0.5$.**
Fig. (3-41) shows the RPS for $\epsilon = 0.5$ and $\delta$ is ranging from zero to 25 pixels using 1, 2, 3, and 4 vanes for a binary star of separation (2,2).

Fig(3-41). RPS as a function of $\delta$ for type (A-1), $\epsilon = 0.5$. 
The power spectrums of an image of binary star using type (A-2) of spiders, separation (2, 2), are then shown in Fig. (3-42).

![Power spectrums of an image of binary star using type (A-2) of spiders, separation (2, 2)](image)

**Fig. (3-42). Images of binary star and their Power spectrum for type (A-2), separation (2, 2), $\varepsilon = 0.25$, $\delta = 3$.**

- **a-** Image of binary star with central obstruction and 3 vanes.
- **b-** Image of binary star of ordinary aperture (without obstruction and spider).
- **c-** Power spectrum with central obstruction and 3 vanes.
- **d-** Power spectrum of ordinary aperture (without obstruction and spider).
- **e-** Diagonal line through (c).
- **f-** Diagonal line through (d).
The power spectrums of an image of a binary star using type (B-2) of spiders, separation (2,2), $\epsilon = 0.25$, $\delta = 5$ pixels and the curve of spider $= 90^\circ$ are then shown in Fig. (3-43).

Fig. (3-43). Images of binary stars and their Power spectra for type (B-2), separation (2,2), $\epsilon = 0.25$, $\delta = 5$.

- **a-** Image of binary star with central obstruction and 4 curved vanes.
- **b-** Image of binary star of ordinary aperture (without obstruction and spider).
- **c-** Power spectrum with central obstruction and 4 curved vanes.
- **d-** Power spectrum of ordinary aperture (without obstruction and spider).
- **e-** Diagonal line through (c).
- **f-** Diagonal line through (d).
The power spectrums of an image of a binary star using type (B-2) of spiders, separation (2,4), $\epsilon = 0.25$, $\delta = 5$ pixels and the curve of spider = $90^\circ$ are then shown in Fig. (3-44).

![Fig. (3-44). Images of binary stars and their Power spectra for type (B-2), separation (2,4), $\epsilon = 0.25$, $\delta = 5$.](image)

- **a-** Image of binary star with central obstruction and 4 curved vanes.
- **b-** Image of binary star of ordinary aperture (without obstruction and spider).
- **c-** Power spectrum with central obstruction and 4 curved vanes.
- **d-** Power spectrum of ordinary aperture (without obstruction and spider).
- **e-** Diagonal line through (c).
- **f-** Diagonal line through (d).
The power spectrums of an image of a binary star using type (B-2) of spiders, separation (2,10), $\varepsilon = 0.25$, $\delta = 5$ pixels and the curve of spider = $90^\circ$ are then shown in Fig. (3-45).

Fig. (3-45). Images of binary stars and their Power spectra for type (B-2), separation (2,10), $\varepsilon = 0.25$, $\delta = 5$.

a- Image of binary star with central obstruction and 4 curved vanes.
b- Image of binary star of ordinary aperture (without obstruction and spider).
c- Power spectrum with central obstruction and 4 curved vanes.
d- Power spectrum of ordinary aperture (without obstruction and spider).
e- Diagonal line through (c).
f- Diagonal line through (d).
Fig. (3-46) illustrates the average power spectrum of a binary star of separation $(2,2)$, $\varepsilon = 0.25$, using type (B-2) of spiders, the curve of spiders $= 90^\circ$ and $\delta$ is ranging from zero to 15 pixels using 1, 2, 3, and 4 vanes.

Fig (3-46). Average Power spectrum as a function of $\delta$ for type (B-2), separation $(2,2)$, $\varepsilon = 0.25$.

Fig (3-47) describes RPS for a binary star of separation $(2,2)$, $\varepsilon = 0.25$, using type (B-2) of spiders, the curve of spiders $= 90^\circ$ and $\delta$ is ranging from zero to 15 pixels using 1, 2, 3, and 4 vanes.

Fig(3-47). RPS as a function of $\delta$ for type (B-2), separation $(2,2)$, $\varepsilon = 0.25$. 
Fig. (3-48) shows the average power spectrum for a binary star of separation (2,4), \( \varepsilon = 0.25 \), using type (B-2) of spiders, the curve of spiders = 90° and \( \delta \) is ranging from zero to 15 pixels using 1, 2, 3, and 4 vanes.

![Diagram showing power spectrum as a function of \( \delta \) for type (B-2), separation (2,4), \( \varepsilon = 0.25 \).](image)

**Fig (3-48). Average Power spectrum as a function of \( \delta \) for type (B-2), separation (2,4), \( \varepsilon = 0.25 \).**

Fig. (3-49) shows RPS for a binary star of separation (2,4), \( \varepsilon = 0.25 \), using type (B-2) of spiders, the curve of spiders = 90° and \( \delta \) is ranging from zero to 15 pixels using 1, 2, 3, and 4 vanes.
Fig (3-49). RPS as a function of $\delta$ for type (B-2), separation (2,4), $\epsilon = 0.25$.

Fig. (3-50) shows the average power spectrum for a binary star with separation (2,10), $\epsilon = 0.25$, using type (B-2) of spiders, the curve of spiders = $90^\circ$ and $\delta$ is ranging from zero to 15 pixels using 1, 2, 3, and 4 vanes.

Fig (3-50). Average Power spectrum as a function of $\delta$ for type (B-2), separation (2,10), $\epsilon = 0.25$. 
Fig. (3-51) presents RPS for a binary star with separation (2,10), \( \varepsilon = 0.25 \), using type (B-2) of spiders, the curve of spiders = 90° and \( \delta \) is ranging from zero to 15 pixels using 1, 2, 3, and 4 vanes.

![Graph showing RPS as a function of \( \delta \) for type (B-2), separation (2,10), \( \varepsilon = 0.25 \).](image)

Fig(3-51). RPS as a function of \( \delta \) for type (B-2), separation (2,10), \( \varepsilon = 0.25 \).
Chapter Four

Conclusions and Future work

The main conclusions that could be drawn from this study are listed below:

1- The vane introduces raw artifacts in the frequency components and two vanes introduce raw and column artifacts. As the number of vanes increases, the contributions of artifacts remain in the same raw and column.

2- The average PSF value for line vanes increases as $\delta$ increases. This leads to the reduction in the resolution. As the number of vanes increases, this value increases sharply and more artifacts are created as shown in Fig. (3-14).

3- As $\epsilon$ increases, the average MTF values reduce significantly. This is because the build up of ripples around the central spike of the PSF [see Fig.(3-15)].

4- Imaging with spiders of type (A-2), the artifacts takes different shape [see Fig.(3-22b)].The energy of central spike of the PSF reduces significantly. This lead to reduce the low frequency components of the MTF. There is no significant difference between this type of spiders and the corresponding (A-1) type in terms of the actual effects on the PSF and MTF.

5- Imaging with curved spider (type B), there is no significant artifacts are noticed in the PSF. The energy of the central spikes remains unchanged even when the number of vanes increases. The effect of these type of spiders on the average PSF and MTF values are insignificant. This leads to high resolution.
6- The power spectra of binary stars show no significant difference between the results of type (A-1) and (A-2). The artifacts that created by these types are completely different but the actual effects on the high frequency components are nearly the same.

7- The RPS is less in terms of spider of type (B). This is so important in terms of maintaining the contrast in the high frequency components. As the separation of binary star increases, the value of RPS goes to the diffraction limited value.

**Future work**

It is so important to use curved spider in optical astronomical imaging in the presence of atmospheric turbulence. This is very important study to investigate the behavior of PSF and MTF in presence of such vanes arranged in type (B).
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الخلاصة

إن الأذرعر التي تحمل المرأة الثانوية تمنع هذه المرأة من الاهتزاز وتعمل على ميلان هذه المرأة باي زاوية يمكن استخدامها.

إن تأثير هذه الأذرعر هو إضافة نتاكات في صور التلسكوبات البصرية و التي تؤدي إلى ظهور معلومات إضافية غير مرغوب فيها.

إن الادبيات في هذا الموضوع في حدود التحليل الكمي لهذه التأثيرات غير مغطى بصورة جيدة و عليه فان دراستنا تضمنت التحليل الكمي لتاثير أنواع مختلفة من هذه الأذرعر الخطية والعنكبوتية عن تفاصيل نموذج الحيوان المسجل من تصوير مصدر نفطي. أن الدراسة شملت أيضاً تأثير هذه الأنواع من الأذرعر على النجوم الثنائية.

النتائج أثبتت أن استخدام أذرعر على شكل منحنى وزوايا مختلفة يكون تأثيره قليل جداً على نموذج الحيوان ومقارنة مع الأنواع الأخرى من الأذرعر.
تأثيرات عنكبوت المرأة الثانوية للتلسكوب على التصوير الفلكي البصري

رسالة مقدمة إلى
كلية العلوم
جامعة بغداد
كجزء من متطلبات نيل درجة الماجستير
علوم في الفلك

من قبل
عمر فاروق يوسف

إشراف
د. علي طالب

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